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Letter to the Editor

A comment on the article "A finite element method for simulation of strong and weak discontinuities in solid mechanics" by A. Hansbo and P. Hansbo [Comput. Methods Appl. Mech. Engrg. 193 (2004) 3523–3540]

Recently, Hansbo and Hansbo [1] have presented an interesting method for modeling arbitrary strong and weak discontinuities, including an a-priori error analysis. The method offers many interesting possibilities. We show here that the kinematic decomposition in their method is equivalent to the one in the extended finite element method (XFEM) by Moës et al. [2] and Belytschko and Black [3]. This conclusion follows from simple algebraic manipulation.

Consider a piecewise displacement decomposition, presented in Ref. [1, Section 3.1]. If a partition of the discretized domain for one particular element is $\Omega = \Omega^+ \oplus \Omega^-$ with + and – denoting the two particular sides of the cracked clement, the we rewrite the authors' displacement decomposition as

$$\boldsymbol{u}(\boldsymbol{X}) = \begin{cases} \boldsymbol{u}_1(\boldsymbol{X}) & \text{in } \Omega^+ \\ \boldsymbol{u}_2(\boldsymbol{X}) & \text{in } \Omega^- \end{cases} \tag{1}$$

with $\boldsymbol{X} \in \Omega$ denoting the position of a particular point. Introducing the Heaviside function $H(\boldsymbol{X})$ such as $H(\boldsymbol{X}) = 0$ for $\boldsymbol{X} \in \Omega^-$ and $H(\boldsymbol{X}) = 1$ for $\boldsymbol{X} \in \Omega^+$ we can write (1) more concisely,

$$\boldsymbol{u}(\boldsymbol{X}) = H(\boldsymbol{X})\boldsymbol{u}_1(\boldsymbol{X}) + [1 - H(\boldsymbol{X})]\boldsymbol{u}_2(\boldsymbol{X}). \tag{2}$$

If we make use of the nomenclature in [4] and introduce the sign function $s(\boldsymbol{X}) = 2H(\boldsymbol{X}) - 1$ and let $s_K = s(\boldsymbol{X}_K)$ with $\boldsymbol{X}_K$ being the position of node K, then the XFEM displacement decomposition for elements not containing a crack tip is as follows:

$$\boldsymbol{u}(\boldsymbol{X}) = \sum_{K=1}^{N} N_K(\boldsymbol{X})\boldsymbol{u}_K + \frac{1}{2}\sum_{K=1}^{N} N_K(\boldsymbol{X})(s(\boldsymbol{X}) - s_K)\boldsymbol{u}_K^{\star}, \tag{3}$$

where N is the number of nodes in a given element, N_K is the shape function corresponding to node K and $\boldsymbol{u}_K$ and $\boldsymbol{u}_K^{\star}$ are the nodal degrees of freedom corresponding to the standard and enriched displacement, respectively. Eq. (3) can be re-written as

$$\boldsymbol{u} = \sum_{K=1}^{N} N_K\boldsymbol{u}_K + \sum_{K=1}^{N} N_K(H - H_K)\boldsymbol{u}_K^{\star}. \tag{4}$$

If, in (4), we replace $\boldsymbol{u}_K$ by $\boldsymbol{u}_K^{\circ} + H_K\boldsymbol{u}_K^{\star}$ it follows that (dropping the independent variables):

$$\boldsymbol{u} = \sum_{K=1}^{N} N_K\boldsymbol{u}_K^{\circ} + \sum_{K=1}^{N} N_K H\boldsymbol{u}_K^{\star} \tag{5}$$

and if we introduce $\boldsymbol{u}_K^1 = \boldsymbol{u}_k^{\circ} + \boldsymbol{u}_K^{\star}$ and $\boldsymbol{u}_K^2 = \boldsymbol{u}_k^{\circ}$ then it follows:

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$$\boldsymbol{u} = H \underbrace{\sum_{K=1}^{N} N_K \boldsymbol{u}_K^1}_{\boldsymbol{u}_1} + (1 - H) \underbrace{\sum_{K=1}^{N} N_K \boldsymbol{u}_K^2}_{\boldsymbol{u}_2}, \tag{6}$$

which is the same decomposition as (2). This result shows that the basis functions in [1] are simply a linear combination of the XFEM basis.

We would like to point out that each of these formulations has advantages in particular problems. Thus XFEM can easily handle partially cracked elements, which is more awkward with the Hansbos' formulation. On the other hand, the Hansbos' formulation can more easily treat nodal degrees of freedom that are not additive, such as directors in shell analysis.

In [1], the authors proposed an extension of Nitsche's method capable of dealing with all spectrum of *constant* interface compliance. A noteworthy result of the Hansbos' article, the bound on the error norm, is a consequence of the specific interface law proposed and of a modified weak form of equilibrium. It can be concluded that if the same modified weak form is employed, the conclusions in [1] hold in the context of the extended finite element method.

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Pedro M.A. Areias
Ted Belytschko
Department of Mechanical Engineering
Northwestern University
2145 Sheridan Road, Evanston
IL 60208-3111, USA
Tel.: +1 847 491 4029; fax: +1 847 491 4011
E-mail address: tedbelytschko@northwestern.edu (T. Belytschko)